Extending an SVAR Model of the Australian Economy*

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Abstract

Dungey and Pagan (2000) present an SVAR model of the Australian economy which models macro-economic outcomes as transitory deviations from a deterministic trend. In this paper we extend that model in two directions. Firstly, we relate it to an emerging literature on DSGE modelling of small open economies. Secondly, we allow for both transitory and permanent components in the series and show how this modification has an impact upon the design of macroeconomic models.

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1 Introduction

The interaction of theory and evidence is a crucial part of applied macroeconomic research. Empirical models should reflect theoretical advances, and theoretical models should adapt to known empirical facts. This paper contributes to the interface by considering the linkages between New Keynesian (NK) and Dynamic Stochastic General Equilibrium (DSGE) theoretical models and the empirical evidence from Structural VARs (SVAR).

In the 1990s we built a SVAR model for Australia - Dungey and Pagan (2000). The focus in that paper was on data coherency; although theoretical ideas formed the basis for empirical restrictions they were applied rather loosely. This paper extends the previous work by paying attention to the consistency of the empirical framework used there with the theoretical properties of NK and DSGE models. The model developed in this paper also formally incorporates the non-stationary and co-integrating properties of the data, interacting these data characteristics with model design. The result is a new model which is a hybrid of the theoretical and empirical approaches, denoted here as H2007.

The Dungey and Pagan (2000) model of the Australian economy was one of the early papers to incorporate the small open economy assumption into a SVAR(p) macroeconomic model - the other being Cushman and Zha (1997). The international sector was represented by US GDP, the real Dow-Jones Index, the terms of trade, exports and the real US interest rate. The domestic economy was represented by the real All-Ordinaries Index, Gross National Expenditure, GDP, the inflation rate, the policy interest rate instrument and the real exchange rate. In this paper we first strengthen the small open economy aspect of the model by conditioning the SVAR in the domestic variables on the exogenous international variables. The resulting model, denoted SVARX2000, forms the benchmark model for this paper, and its domestic properties are virtually indistinguishable from those in Dungey and Pagan (2000).

The paper then continues to pay specific attention to potential non-stationarity in the data. Theoretical DSGE and NK models are often designed to allow for non-stationarity by converting the models to a "gap" form wherein the data has been transformed so as to remove any permanent components in the series. Empirical models also work with transformed non-stationary variables, mainly through Structural Vector Error Correction Models (SVECMs). Here we show that care has to be exercised in the conversion process of theoretical models in order that the two methods are equivalent. The H2007 model developed in this paper has a theoretical orientation but also reacts to the nature of the data, specifically the evidence for co-integration among the variables.

Non-stationarity in data is not the only difficulty arising when one attempts to reconcile empirical and theoretical models. It may be that more variables enter empirical than theoretical models, more complex dynamics may be in the former than the latter, and the rational expectations hypothesis that is the mainstay of theoretical models may not be empirically supported. Our paper addresses all these issues when constructing our new model H2007. In regards to the second of these, it is worth mentioning that,although many DSGE models do have a SVAR representation in terms of all the variables entering them, in practice some of those variables are unobserved, and so the representation in terms of observed variables is generally a Structural Vector Autoregressive Moving Average (SVARMA) process. Consequently, in terms of any empirical model formulated in a smaller set of variables, the data will need to be approximated by a SVAR of higher order than would have been implied by a DSGE model expressed as an SVAR in its total set of variables. Examples of recent work in which this issue has arisen are Del Negro, Schorfheide, Smets and Wouters (2005), Christiano, Eichenbaum and Vigfusson (2006) and Chari, Kehoe and McGrattan (2007). The problem is an old one though, and was first identified in Zellner and Palm (1974) and

Wallis (1977). A discussion of it is given in Fry and Pagan (2005).[1]

In using the new hybrid model, H2007, many of the responses of the Australian economy to temporary shocks are very similar to the those in SVARX2000. However, because there are permanent shocks in H2007 and only transitory ones in SVARX2000, there must be quite different long run outcomes for the permanent shocks. It is not possible to formally test the "significance" of these differences except to say that any such "test" would be a function of the test for co-integration. If there is co-integration then there must be shocks in the system that are different in SVARX2000 to H2007. Accepting the co-integration hypothesis therefore has important effects upon model design and the impact of shocks at long horizons. But it may also have them at shorter horizons and this seems to be the case in our work. Most importantly the results we obtain suggest that the previous model overstated the impact of monetary policy shocks on macroeconomic activity.

The paper proceeds as follows. Section 2 considers the theoretical framework for designing small open-economy models with stationary data. Section 3 then extends this to non-stationary data, highlighting potential difficulties in reconciling the "gaps" representation used in many theoretical models with the SVECM form. The new hybrid model, H2007, is developed in Section 4, while Section 5 illustrates the differences in outcomes from modelling the Australian economy with H2007 rather than SVARX2000. Section 6 concludes.

2 Model Design with I(0) Variables

The New Keynesian Open Economy Model has become an important part of the modern macroeconomic tool box. Examples include Svensson (2000), Justiniano and Preston (2006), Lubik and Schorfheide (2007), Gali and Monacelli (2005), Giordani (2004), Nimark(2007) and Buncic and Melecky (2008).

[1] Most DSGE models imply a VAR(2) in all their variables so both SVARX2000 and H2007 use a higher order VAR.

Although a number of these models are quite large, the basic model can be summarized in the following five equations.

$$\xi_t = \eta_1 \xi_{t-1} + \eta_2 E_t(\xi_{t+1}) + \eta_3(r_t - E_t \pi_{t+1}) \quad (1)$$

$$y_t = a_1 \xi_t + a_2 x_t - a_3 m_t \quad (2)$$

$$\pi_t = \eta_4 \pi_{t-1} + \eta_5 E_t(\pi_{t+1}) + \eta_6(y_t - y_t^n) + \eta_7 E_t \Delta\zeta_{t+1} + \eta_8 \Delta s_t + u_{Pt} \quad (3)$$

$$r_t = \eta_9 r_{t-1} + \eta_{10}(y_t - y_t^n) + \eta_{11}\pi_t + \eta_{12}\zeta_t + u_{rt}, \quad (4)$$

$$\zeta_t = E_t(\zeta_{t+1}) + (r_t - \pi_t) - (r_t^* - \pi_t^*) + u_{\zeta t} \quad (5)$$

In order, these equations comprise an IS curve describing absorption (GNE), a log linearization of the GDP identity, a Phillips curve, an interest rate rule and a real UIP relationship for the exchange rate. The variables are represented by ξ_t as the log of absorption (GNE), r_t as a nominal interest rate, π_t as domestic inflation, y_t as the log of GDP, x_t as the log of exports, m_t as the log of imports, y_t^n as the potential level of output, ζ_t as the real exchange rate (expressed such that an increase in ζ_t represents an appreciation of the domestic currency), s_t as the log of the terms of trade, r_t^* as the foreign nominal interest rate and π_t^* as foreign inflation. If these variables are stationary they are measured as deviations from some steady state values. We return to the nonstationary case a little later. A number of alternative means of dating expectations are possible in the policy rule, as well as a variety of methods for measuring lagged inflation in the Phillips curve, neither of which will fundamentally affect the current discussion.[2] As is standard in the NK and DSGE literature, supply shocks (which incorporate effects from the growth in technology) are represented by a shock term in the Phillips curve, u_{Pt}, monetary policy shocks are designated by u_{rt} in the monetary policy reaction function, and there is a shock to the UIP relationship given by $u_{\zeta t}$.

[2] For example, Giordani (2004) and Berg et al. (2006) replace π_{t-1} in the Phillips curve by the average inflation rate over the past four quarters.

The "IS curve" given in equation (1) describes absorption. A common assumption is that there is no investment, so that ξ_t is consumption and the equation describing it is a standard one found from households' optimizing behaviour and the presence of habit persistence, see for example Gali and Monacelli (2005), Lubik and Schorfheide (2007) and Justiniano and Preston (2006).

The GDP equation stems from a log linearization of the identity involving GDP, GNE and the trade balance, as in Gali and Monacelli (2005). The trade balance is replaced in many models by a combination of foreign output y_t^*, the terms of trade and the real exchange rate, with varying treatment of the law of one price. In this tradition it is possible to write the log-linearized trade balance as

$$a_2 x_t - a_3 m_t = \phi_3 s_t + \phi_4 \zeta_t + \phi_5 y_t^* + \varepsilon_{mt}, \tag{6}$$

where ε_{mt} is a shock reflecting changes in preferences between domestic and imported goods. The GDP equation may then be expressed as

$$y_t = a_1 \xi_t + \phi_3 s_t + \phi_4 \zeta_t + \phi_5 y_t^* + \varepsilon_{mt}. \tag{7}$$

Frequently GNE is eliminated and replaced by GDP when theoretical models are estimated with data. However, as this creates quite a complex structural equation, absorption and output will be kept separate here.

The Phillips curve given in equation (3) is a standard one for an open-economy. Following Gali and Gertler (1999) most theoretical models of inflation are driven by the gap between real marginal cost (real unit labour costs) and the steady state value of these. If the steady state corresponds to fully flexible pricing it is generally possible to replace that gap with the output gap, $y_t - y_t^n$, where y_t^n is the level of output in a flexible price economy. The form of y_t^n depends upon the assumptions made in deriving the model.

The external sector in the model of equations (1) to (5) is represented by foreign output, foreign interest rates and foreign inflation. Exports and the terms of trade will be partly determined by domestic and partly by foreign forces although both are often treated as being externally determined, particularly if the country is a commodity exporter. Hence these five variables can be viewed as strongly exogenous, and constitute observable shocks into the model. In this case solving the model produces a VARX system in GNE, GDP, inflation, the domestic interest rate and the real exchange rate. It is immediately clear that a VARX representation of this NK model involves the same variables as SVARX2000, with the exception that SVARX2000 has two extra variables representing external and internal real asset prices.

As mentioned earlier, a potential impediment to relating empirical and theoretical models is the role of rational expectations. Sometimes it is more appropriate from an empirical perspective to work with the same information set as theoretical models, but to allow the data to unrestrictedly determine the weights applied to the variables in the set. Moreover, because theoretical models often treat agents as having contemporaneous information available when forming their expectations, it may be empirically more plausible to use an alternative in which the appropriate information set comprises the current observed exogenous variables, lagged values of any endogenous variable, and some shocks. This is done in H2007.

How this assumption affects model design is now considered in connection with the IS curve in the NK model. The expectation of future GNE will be taken to be a function of the observable endogenous variables at $t-1$, denoted z_{t-1}, and current observable exogenous variables, w_t. Assuming that the agent knows the log of the current level of technology, a_t, but that other non-serially correlated shocks are set to zero when forming expectations, then the expectations of future GNE are generated by

$$E_t(\xi_{t+1}) = z'_{t-1}\lambda_1 + w'_t\lambda_2 + \psi a_t, \tag{8}$$

and the IS equation takes the form

$$\xi_t = \eta_1\xi_{t-1} + \eta_2(z'_{t-1}\lambda_1 + w'_t\lambda_2 + \psi a_t) + \eta_3(r_t - E_t\pi_{t+1}). \tag{9}$$

The expected level of future inflation does not involve the level of technology but rather its expected rate of change. In the simplest case, $\Delta a_t = \varepsilon_{at}$ is white noise, giving an expected rate of change of zero, so that $E_t\pi_{t+1}$ is simply a function of z_{t-1} and w_t i.e $E_t\pi_{t+1} = z'_{t-1}\gamma_1 + w'_t\gamma_2$, leading to the IS curve

$$\xi_t = z'_{t-1}\phi_1 + w'_t\phi_2 + \eta_3 r_t + \psi_3 a_t. \tag{10}$$

If a_t is serially correlated the absorption equation would involve further lags of the variables. Similar substitutions will produce a set of structural equations for the NK system involving z_{t-1} and w_t variables as follows,

$$\begin{aligned}
\xi_t &= z'_{t-1}\phi_1 + w'_t\phi_2 + \eta_3 r_t + \psi_3 a_t & (11)\\
y_t &= a_1\xi_t + \phi_3 s_t + \phi_4\zeta_t + \phi_5 y^*_t + \varepsilon_{mt} & (12)\\
\pi_t &= z'_{t-1}\phi_6 + w'_t\phi_7 + \eta_6(y_t - y^n_t) + \eta_8\Delta s_t + u_{Pt} & (13)\\
r_t &= \eta_9 r_{t-1} + \eta_{10}(y_t - y^n_t) + \eta_{11}\pi_t + \eta_{12}\zeta_t + u_{rt} & (14)\\
\zeta_t &= z'_{t-1}\phi_6 + w'_t\phi_7 + (r_t - \pi_t) - (r^*_t - \pi^*_t) + u_{\zeta t}. & (15)
\end{aligned}$$

3 Model Design with I(1) Variables

When some of the data series are unit root processes this must be explicitly accounted for in both the theoretical and empirical structures. Simple DSGE models presume that the $I(1)$ variables are y_t, y^n_t and ξ_t and that there is one permanent component among them which is associated with the log level of technology a_t i.e. $\Delta a_t = \varepsilon_{at}$. Hence there will be co-integration between y_t, ξ_t and y^n_t. The $I(1)$ variables are rendered stationary by assuming that the log of technology a_t is an $I(1)$ process, $\Delta a_t = \varepsilon_{at}$, and then by defining $\tilde{\xi}_t = \xi_t - a_t, \tilde{y}_t = y_t - a_t$, $\tilde{y}^n_t = y^n_t - a_t$. The original variables in the system described in Section 2 are now replaced with the tilde values. In the case of the IS equation (1) with $\eta_1 + \eta_2 = 1$ and ε_{at} being white noise, this produces

$$\begin{aligned}\tilde{\xi}_t &= \eta_1\tilde{\xi}_{t-1} + \eta_2 E_t(\tilde{\xi}_{t+1}) + \eta_3(r_t - E_t\pi_{t+1}) \\ &\quad -a_t + \eta_1 a_{t-1} + \eta_2 E_t(a_{t+1}) \\ &= \eta_1\tilde{\xi}_{t-1} + \eta_2 E_t(\tilde{\xi}_{t+1}) + \eta_3(r_t - E_t\pi_{t+1}) + (\eta_2 - 1)\Delta a_t. \qquad (16)\end{aligned}$$

The presence of a single $I(1)$ factor a_t in the system means that (ξ_t, y_t) and (y_t, y_t^n) are co-integrating pairs, since there are three $I(1)$ variables driven by a single common permanent component. Indeed the co-integrating vectors in this case are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. It is often the case that y_t^n is taken to be a linear function of a_t so that the output gap $y_t - y_t^n$ equals the transitory component of y_t.[3] This points to the need to extract the permanent components of any $I(1)$ series.

Many empirical applications of NK and DSGE models do not work with the latent variables $\tilde{\xi}_t$ and $\tilde{y}_t$ but rather filter the data on ξ_t and y_t to remove a permanent component and the use the transitory components as the proxies for $\tilde{\xi}_t$ and $\tilde{y}_t$.[4] We now examine the impact of such an approach, and, in particular, whether it would respect the co-integrating relationships in the data.

Assume that the n variables of the $SVAR$, z_t, may contain both $I(1)$ and $I(0)$ variables and the $I(1)$ variables may be co-integrated. For expositional convenience we adopt an SVECM(2) form - higher orders simply lead to more complex notation.

$$B_0\Delta z_t = \alpha^*\beta' z_{t-1} + B_1\Delta z_{t-1} + \varepsilon_t \qquad (17)$$

$$\implies \Delta z_t = \alpha\beta' z_{t-1} + C_1\Delta z_{t-1} + e_t, \qquad (18)$$

where $\alpha = B_0^{-1}\alpha^*$ and $e_t = B_0^{-1}\varepsilon_t$. To allow for both $I(1)$ and $I(0)$ variables to be present in z_t, and yet still use the VECM form in (18), it is simplest

[3] Suppose we have a Cobb-Douglas production function and factors are set to their flex-price levels. Then $Y_t = \bar{K}^\alpha(A_t\bar{L})^{1-\alpha}$ and so y_t will be a linear function of a_t.

[4] The removal of a permanent component is regarded as removing supply side effects.

to define some pseudo co-integrating vectors in the following way. Suppose the first n_1 variables in the system are $I(1)$ and have $r_1 < n_1$ co-integrating vectors (β_1) between them. To account for the remaining $n-n_1$ $I(0)$ variables simply define β as

$$\beta = \begin{bmatrix} \beta_1 & 0 \\ 0 & -I_{n-n_1} \end{bmatrix}. \tag{19}$$

The equations for the $I(0)$ variables will then have $-\alpha_j z_{j,t-1}$ appearing in the equation of (18) that has Δz_{jt} as its dependent variable, meaning that z_{jt} will be $I(0)$ provided $|\alpha_j| < 1$.

In (18) there will be $n_1 - r_1$ common permanent components of the form

$$\Delta\tau_{1t} = \eta_t, \tag{20}$$

where η_t is white noise. The permanent component of z_{1t}, z_{1t}^p, can be written as $z_{1t}^p = J\tau_{1t}$, where $J = \beta_\perp(\alpha_\perp' \Gamma \beta_\perp)^{-1}\alpha_\perp' B_0^{-1}\varepsilon_t$ and $\beta'\beta_\perp = 0$, $\alpha_\perp'\alpha = 0$, $\Gamma = I - C_1$. This is the Beveridge-Nelson method for extracting permanent components and is the appropriate model-consistent filter.[5]

In the existing literature, when only $I(1)$ variables are present the SVECM form is often transformed to an SVAR form involving $n - r$ of the Δz_t and the r ECM terms $\beta' z_{t-1}$, for example Gali (1992, 1999). Although the ECM terms are sometimes vaguely referred to as "gaps", they are not the same "gaps" that appear in NK/DSGE models. To see this we transform the SVECM in (17) by adding and subtracting functions of z_t^p and its lags and then use the property that $\beta' z_t^p = 0$ to get

$$B_0\Delta(z_t - z_t^p) = \alpha^*\beta' z_{t-1} + B_1\Delta(z_{t-1} - z_{t-1}^p) + B_1\Delta z_{t-1}^p - B_0\Delta z_t^p + \varepsilon_t. \tag{21}$$

(21) shows that "gaps" (transitory components) are actually combinations of the lagged ECM terms and changes in the permanent components. It is clear from this decomposition that any SVAR using gaps will also need to

[5] Other formulations such as Hodrick-Prescott would be inappropriate and will generally lead to inconsistent estimators of the model parameters, as discussed in Fukac and Pagan (2008).

incorporate Δz_t^p and its lags in order to be compatible with the data. A model which excludes these terms will induce extra elements into the error terms of the equations, and thereby violate a common assumption in SVAR work that the shocks have no serial correlation. The problem essentially arises due to the unobservable nature of the common permanent components in most theoretical models. The attempt to circumvent this lack of observability via filtering of the data creates the difficulties outlined above.

4 Development of the H2007 Model

The SVARX2000 model consists of the 6 endogenous domestic variables of Dungey and Pagan (2000) and the 5 exogenous international variables. The structure of the domestic sector is the same as in Dungey and Pagan (2000).[6] The data were updated to 2006Q4 and SVARX2000 re-estimated. The results were quite close to those reported with the original data. Relating SVARX2000 to the generic NK model outlined above, we find that nine of the variables - domestic inflation, the real exchange rate, exports, the terms of trade, domestic and foreign interest rates, foreign output, domestic GDP and GNE - appear in both. The two extra variables in the SVARX2000 system are foreign and domestic real equity prices, and these aim to augment the explanation of the investment component of GDP and GNE - see also de Roos and Russell (1999).[7] The data definitions for each variables are the same as in Dungey and Pagan (2000) but, for completeness, are presented in Appendix A .

In SVARX2000 the data were detrended and demeaned prior to estimation, a feature also of models such as Lubik and Schorfheide (2005) and Buncic and Melecky (2008) for example. However in H2007 only real vari-

[6] The only alteration to the Dungey and Pagan specification is the inclusion of a once-off dummy for the introduction of GST in the third quarter of 2000.

[7] The real interest rate represents a cost term whereas the return to investment is captured by real equity prices.

ables are implicitly being detrended through their presence in the ECM terms of the VECM.[8] There are intercepts in the structural equations and when ECM terms are not present these can represent a de-trending operation when the equation is for an$I(1)$ variable. Of course, as in any regression equation, the intercept also adapts to the mean values of the $I(0)$ variables. Nominal variables such as real equity prices, inflation and the interest rate are left in levels. This is an attractive feature.

As a first step to developing a model which incorporates insights from both the empirical perspective of SVARX2000 and the theoretical NK model of Section 2, it is useful to explicitly explore the differences in structure between them. This is done by comparing the equations of the SVARX2000 system set out in Table 1 with the NK model equations in (11)-(15). Three important differences occur. First, since SVARX2000 has a triangular structure and NK has not, some contemporaneous influences in the latter model do not appear in the former. Specifically η_3, ϕ_4 and η_{12} are zero in SVARX2000 i.e. there are no contemporaneous effects of the real interest rate on GDP, and of the real exchange rate on GDP and interest rates. The first has a long history of use in Australian empirical work e.g. Gruen, Romalis and Chandra (1997), the second of these is supported by Buncic and Melecky (2008) who find the effect to be very small, and the last is in agreement with Lubik and Schorfheide (2005). In all instances lagged exchange rates and interest rates have an effect, so it is an issue of the timing of these rather than their existence.

Second, a number of relationships which are included in SVARX2000 have no counterparts in the NK model. These are: (i) The inclusion of real equity prices, q_t and q_t^*, in SVARX2000, as already discussed, (ii) A direct effect of exports on GDP as the trade balance has been substituted out in the NK model, (iii) SVARX2000 utilizes a GNE rather than GDP gap, and

[8]These would not be present if the variables co-trended with the same weights as the co-integrating vectors but that is often not the case in applied work.

Table 1:
A comparison of the form of the SVARX2000 and H2007 models.

	SVARX2000	H2007
data	detrended, zero mean all $I(0)$	data not detrended mix of changes in $I(0)$ and $I(1)$ variables and two co-integrating vectors $ecm_{1t} = y_t - 0.31y_t^* - 0.07x_t$ $-0.61\xi_t - 0.0005t - 1.53$ $ecm_{2t} = y_t - 0.64\xi_t + 0.21\varsigma_t$ $-0.0026t - 4.58$
lags	3 lags in levels	3 lags when in levels, 2 in VECM form
Internat. Sector	$y_t^*, (r_t^* - \pi_t^*), s_t,$ x_t, q_t^*	$\Delta y_t^*, (r_t^* - \pi_t^*), s_{t-i},$ $\Delta x_t, q_t^*$ Δy_t^* and Δx_t as $AR(2)$
Eqn		
q_t	$q_t = f\{y_{t-i}^*, s_{t-i}, q_{t-i}^*,$ $(r_{t-k}^* - \pi_{t-k}^*), q_{t-k}, \xi_{t-k},$ $y_{t-k,}\pi_{t-k}, r_{t-k}, \varsigma_{t-k}\}$	$q_t = f\{\Delta y_{t-j}^*, s_{t-i}, q_{t-i}^*, (r_{t-k}^* - \pi_{t-k}^*),$ $q_{t-k}, \Delta\xi_{t-h}, \Delta y_{t-h,}\pi_{t-k}, r_{t-k}, \Delta\varsigma_{t-h},$ $ecm_{1,t-1}, ecm_{2,t-1}\}$
ξ_t	$\xi_t = f\{s_{t-i,}q_{t-i}, \xi_{t-k}, \pi_{t-k},$ $r_{t-g}, \varsigma_{t-k}\}$	$\Delta\xi_t = f\{\Delta s_{t-i,}\Delta q_{t-i}, \Delta\xi_{t-k}, \Delta\pi_{t-k},$ $\Delta r_{t-g}, \Delta\varsigma_{t-h}\}$
y_t	$y_t = f\{\xi_{t-i}, x_{t-i}, q_{t-i},$ $y_{t-i}^*, \pi_{t-k}, r_{t-g}, \varsigma_{t-k}\}$	$\Delta y_t = f\{\Delta y_{t-j}^*, s_{t-i}, \Delta x_{t-j}, q_{t-j}, \Delta\xi_{t-j},$ $\pi_{t-k}, r_{t-g}, \Delta\varsigma_{t-h}, ecm_{1,t-1,}ecm_{2,t-1}\}$
π_t	$\pi_t = f\{\xi_{t-i}, s_{t-i}, \pi_{t-k}, \varsigma_{t-k}\}$	$\pi_t = f\{\Delta\xi_{t-j}, s_{t-i}, \pi_{t-k}, \Delta\varsigma_{t-h},$ $ecm_{1,t-1}, ecm_{2,t-1}\}$
r_t	$r_t = f\{\xi_{t-i}, \pi_{t-i}, r_{t-k}, \varsigma_{t-k}\}$	$r_t = f\{\Delta\xi_{t-j}, \pi_{t-i}, r_{t-k}, \Delta\varsigma_{t-h,}$ $ecm_{1,t-1}, ecm_{2,t-1}\}$
ς_t	$\varsigma_t = f\{y_{t-i}^*, s_{t-i}, (r_{t-i}^* - \pi_{t-i}^*),$ $q_{t-i}^*, q_{t-i}, \xi_{t-i,}y_{t-i}, \pi_{t-i},$ $r_{t-i}, \varsigma_{t-k}\}$	$\Delta\varsigma_t = f\{\Delta y_{t-j}^*, s_{t-i}, (r_{t-i}^* - \pi_{t-i}^*),$ $q_{t-i}^*, \Delta x_{t-j}, q_{t-i}, \Delta\xi_{t-j,}\Delta y_{t-j}, \pi_{t-i},$ $r_{t-i}, \Delta\varsigma_{t-j}, ecm_{1,t-1}, ecm_{2,t-1}\}$

Subscript Notation: $i = 0, 1, 2, 3,$ $j = 0, 1, 2,$ $k = 1, 2, 3,$ $h = 1, 2,$ $g = 2, 3$
Variable Definitions: y_t^* foreign output, s_t terms of trade, $r_t^* - \pi_t^*$ foreign real interest rate, q_t^* foreign real share price, x_t export volume, ξ_t absorption (GNE), q_t domestic real share price, y_t domestic output, π_t inflation, r_t interest rate, ς_t real exchange rate such than an increase is an appreciation in the domestic currency.

(iv) real UIP implies precise coefficient restrictions that are not imposed in SVARX2000. In relation to "gap measurement" the reasons for working with GNE rather than GDP were laid out in Dungey and Pagan (2000), where it was argued that GNE potentially provides a better measure of demand in a small open economy - the NK framework of Section 2 provides further support for that particular innovation.

A major objective of this paper is to accommodate the non-stationary and co-integrating properties of the data. In line with most empirical work we regard foreign output, exports, GNE and GDP as $I(1)$, while inflation, q-ratios and interest rates are $I(0)$. The real exchange rate is a more difficult case. When there are differences in technology between countries, the real exchange rate may have to adjust in order to keep the current account as a certain fraction of GDP. In later analysis we show that it is difficult to reject three common permanent components in the system and, if we interpret two of these as representing global and local technology, the real exchange rate must have a permanent component. The ADF (3) test for whether it has a unit root is -1.47 versus a critical value of -3.46, although it should be noted that the point estimate of the lagged variable is only 0.93. Despite this point estimate, and in line with the formal test, our decision was to treat it as an $I(1)$ process.

In testing for co-integration between the $I(1)$ variables, exports and foreign output are treated as exogenous, and the tests augmented with the $I(0)$ exogenous variables $(s_t, (r_t^* - \pi_t^*)$ and $q_t^*)$. The max and trace test statistics point to there being two co-integrating vectors with test statistics of 24.5 and 19.2 versus 95% critical values of 23.4 and 18.1 respectively.[9] Two restrictions are then needed to identify these co-integrating vectors. In both instances one restriction comes from normalizing on GDP. Thereupon, excluding the real exchange rate from the first co-integrating relation identifies it. For the

[9] Computations were done with MFIT5 which exploits the VARX structure in designing these tests. As in the original paper a VAR(3) was the basic model upon which tests were performed.

second vector we excluded both of the foreign exogenous variables. This then results in over-identification. We take the identifying constraint to be the exclusion of exports and test whether USGDP can also be excluded from the vector. A $\chi^2(1)$ test of 0.01 easily accepts the constraint. The identified vectors are given at the top of the right side panel of Table 1. Figure 1 plots the errors corresponding to the first co-integrating vector.

To be consistent with an NK model the GDP equation (12) would imply that the weight on ξ_t in the co-integrating relation between y_t, ξ_t and ζ_t would be one less the import share, while that on ζ_t would be the negative of that value, provided the expenditure elasticity of imports was unity. So we might expect these weights to be close to -.8 and .8, but such a restriction seems to be rejected by the data. Again, this shows a tension between constraints provided by theoretical models and what is in the data. There seems no easy answer to what one does in response to this.

The presence of a co-integrating vector between Australian and US GDP was an integral part of models such as Beechey et al (2000), although more recent versions of that model such as Stone et al (2005) have discarded it. The reason for this is apparent from examining the residuals from a regression of log GDP on the log of US GDP - the Durbin-Watson test of .23 shows large amounts of serial correlation and would indicate a unit root in them. However, if one adds into this regression the log of GNE and the log of exports it rises to .65, and a unit root in the residuals is easily rejected. The co-integrating relationship denoted ecm_1 in Table 1 also holds over the shorter period to 1995/4 (based on the trace test), and the weights would be .57, .05 and .28, which are quite close to the estimates over the longer period. Consequently, GNE clearly contains important information about permanent components.

The existence of two co-integrating vectors among the five variables means that there are three independent permanent components. As observed earlier, in theoretical models there is usually a single permanent component, the level of technology, although Edge et al. (2007) include two components. Most theoretical models assume that domestic and foreign technology share a common $I(1)$ component, with deviations between them being represented by some $I(0)$ factors. The Australian data clearly rejects this idea.[10]

The two exogenous $I(1)$ variables - foreign GDP and exports - will be treated as two of the three permanent components. These then evolve as separate $I(1)$ processes. This may seem unsatisfactory, but variables which do not cointegrate may stay close to one another for sustained periods. Exports might well have a separate permanent component for two reasons. One is that Australian markets for exports are now located mainly in the Asian region, and US GDP is likely to be a poor proxy for Asia. Another is that modelling trade flows often requires gravity models. Since exports and imports grow much faster than domestic (and foreign) GDP, gravity models relate trade flows to a weighted average of foreign plus domestic GDP, where the weights sum to more than unity. Australian export growth has certainly been higher than output growth over much of our sample - see Figure 2.

One remaining permanent component is required.[11] The NK model suggests the shock in the structural equation for GNE. Pagan and Pesaran (2008) show that there should be no ECM term in any structural equation whose shock is permanent. This is true of the GNE equation in H2007 but not the GDP equation[12]. Hence the permanent shock is associated with the GNE equation.

To understand the way in which H2007 relates to SVARX2000 consider a simplified version of a typical equation from the VAR of the latter. The

[10] One possible reason for this is that it is a temporary phenomenon due to domestic productivity increases in the 1990s, although, as this also occurred in the US, the argument is not convincing. Another is that the data are not expressed in per capita form.

[11] Because we treat exports and US GDP as separate univariate components the permanent component of any of the series is easily found using the univariate Beveridge-Nelson decomposition of it.

[12] The reason why it is possible to test whether the ECM term is present or not is the assumption that H2007 is recursive, done to make it comparable to SVARX2000.

simplification is that the system only have three variables (rather than 11) and be a VAR(2) (rather than a VAR(3)). In SVARX2000 a typical equation describing an endogenous variable would have the form

$$
\begin{aligned}
y_{1t} &= a_{11}^{1} y_{1t-1} + a_{12}^{1} y_{2t-1} + a_{13}^{1} y_{3t-1} + a_{11}^{2} y_{1t-2} \\
&\quad + a_{12}^{2} y_{2t-2} + a_{13}^{2} y_{3t-2} + e_{1t},
\end{aligned} \tag{22}
$$

whereas the equivalent equation in H2007 (assuming just one co-integrating relation) is

$$
\Delta y_{1t} = \alpha_1 ecm_{1t-1} + \gamma_{11}^{1} \Delta y_{1t-1} + \gamma_{12}^{1} \Delta y_{2t-1} + \gamma_{13}^{1} \Delta y_{3t-1} + e_{1t}. \tag{23}
$$

So the same levels variables are involved in both representations but the H2007 form (23) is a re-parameterized version of the SVARX2000 one. Note that (23) has a maximum lag in differences that is one less than the order of the levels in (22). It is this comparison that is provided in Table 1. Consequently, as SVARX2000 is a VAR(3), estimation of H2007 is undertaken with two lags of differenced variables for comparability.

Now there are some equations in H2007 whose origin does not agree with the description above. One example would be that, although lagged exports do not appear in the SVARX2000 exchange rate equation, it does appear in the H2007 model via an error correction term. Another would be the fact that lagged ECM terms in the interest rate and inflation equations are needed in H2007 to capture the GNE gap, since, as shown earlier, the GNE gap will be function of changes in variables and ECM terms. But this now introduces lagged variables such as GNE and USGDP into the corresponding H2007 equations, and these were not present in SVAR2000. In the latter only GNE appeared. Consequently, if we had retained the SVARX2000 structure of those equations it would have been necessary to enter GNE in differenced form. Otherwise, the orders of the integration on both sides of the equations would not agree. So in the cases just outlined the H2007 equations cannot be regarded as a re-working of the equivalent SVARX2000 equations, but require

Table 2:
Size of One Standard Deviation Shocks (as percentages in SVARX2000).

variable	shock	variable	shock
y_t^*	0.6%	q_t	7.2%
s_t	1.7%	ξ_t	0.6%
$r_t^* - \pi_t^*$	148bp	y_t	0.5%
x_t	8.0%	π_t	2.2%pa
q_t^*	3.3%	r_t	173bp
		ς_t	2.1%

modification to recognize the fact that certain variables are $I(1)$ and this affects the model design. Note that the GNE equation structural equation in H2007 is expressed with all variables in changes, enforcing the restriction that the shock in this equation is permanent, see Pagan and Pesaran (2008).

5 Empirical Results

As H2007 retains a recursive structure estimation proceeds on an equation by equation basis in Gauss 6.0.[13] The impulse responses are produced along the lines of Appendix B. The sizes of the shocks administered to each model are the same in each case, corresponding to one standard error shocks to the equations in SVARX2000. These are shown in Table 2. A complete set of impulse responses and formal 90% bootstrapped error bands for H2007 are provided in the working paper version, Dungey and Pagan (2008).

5.1 Impulse Responses

H2007 contains both temporary and permanent shocks, while SVARX2000 contains only temporary shocks. One would expect there to be substantial

[13]Codes and data for the estimation of SVARX2000 and H2007 can be obtained from www.dungey.bigpondhosting.com

differences at long horizons between the impulse responses to a shock that is treated as permanent in one model and transitory in the other. However, it may be that the transitory shocks are very persistent and so it may take a long time for any differences to show up. The impact of the transitory shocks in each of the models may or may not differ at short horizons. It emerges that there are a number of cases where the impulses are quite similar.

Impulse Responses to Shocks from Exogenous Variables A shock to foreign output, y_t^* is permanent in H2007 and temporary in SVARX2000. A selection of impulse responses to this shock are shown in Figure 3. Panel (a) shows that the effects on domestic real equity prices, q_t, are relatively similar in the two models, while the effect on domestic GDP, y_t, is permanent in H2007. The response of y_t re-crosses the baseline after almost 5 years in SVARX2000. The effects of the y_t^* shock on inflation, π_t, panel (c), and the interest rate, r_t, (not shown) are smaller in the H2007 model than in SVARX2000. However, in both cases the real interest rate falls by a similar amount and prompts a depreciation in the real exchange rate, ς_t, panel (d). The $I(1)$ nature of the exchange rate means the depreciation is permanent in H2007.

The effects of a foreign preference shock (for domestic goods by foreigners) are seen in the shock to the terms of trade, s_t. In H2007 this leads to an increase in GNE (shown in Figure 4 panel (a)) and GDP. The inflation and interest rate responses for both models are remarkably similar in the first year, panels (b) and (c). As expected, given the known strong relationship between the terms of trade and real exchange rate for Australia, the currency appreciates in response to the shock, panel (d). These effects are all transitory, although in H2007 they can take a considerable time to return to equilibrium.

The effects of shocks to the foreign real interest rate, $(r_t^* - \pi_t^*)$ and foreign real equity prices, q_t^* are consistent with those recorded in Dungey and Pagan

(2000), although the shocks to q_t^* take longer to dissipate. A temporary shock to q_t^* leads to an increase in domestic real equity prices, GNE and GDP (q_t, ξ_t and y_t) and are shown in Figure 5 panel (a-b). The interest rate and inflation responses in H2007 are quite volatile in the first 18 months, just as in SVARX2000, but then become negative and dissipate from there, (panels c-d).

The shock to the export equation, x_t, is permanent in H2007 versus the transitory shock in SVARX2000. The results are shown in Figure 6. A permanent increase in exports results in a small initial rise in q_t, panel (a). The effect of the export shock on GDP is much greater than that on GNE, panels (b-c). Towards the end of the impulse horizon shown the GNE response is negative, possibly because the interest rate has been held high for a relatively long period (panel e). From theoretical models it might be expected that there would only be small changes in GDP but a larger rise in GNE, due to a wealth effect which leads to higher expenditure on imports, balancing out the increased exports. Such a restriction could be enforced. However this is not done here. The effects of a permanent export shock in H2007 are more satisfactory than the temporary export shock in SVARX2000 which resulted in declines in both GDP and GNE (panels b-c). Consequently the inflation and exchange rate responses also differ (panels d and f), in particular showing a long run depreciation of the currency.

Impulse Responses to Shocks From Domestic Variables This section presents results on five of the domestic shocks. The omitted shocks are those to inflation and the real exchange rate, π_t and ς_t. These are particularly difficult to interpret, although they might be regarded as markup and risk premium shocks.[14] In general, the responses to both shocks are similar in the H2007 and SVARX2000 models.

[14] It is worth noting that the perverse output decline in response to an inflation shock seen in the VECM(2) representation of our restrictions in Buncic and Melecky (2008) occurs in neither H2007 nor SVARX2000.

Real equity price, q_t, shocks

The shocks to q_t have a positive impact on GNE and GDP in both the SVARX2000 and H2007 models, see Figure 7 panels (a) and (b) for example. The main differences are in the scale of the responses in GDP, inflation, π_t, and interest rates, r_t (panel b-d).

GNE, ξ_t, shocks

In the NK model the shock to ξ_t represents a domestic technology shock. In the empirical models the shock to ξ_t results in a strong initial positive effect on the levels of both GNE and GDP - see Figure 8, panels (a-b). In H2007, where this is a permanent shock, the effects on GNE and GDP are sustained. In H2007, after 5 years GNE is 0.8 percent above baseline while GDP is 0.5 percent above it consistent with an expansion of incomes and associated rise in imports. In SVARX2000 where the shock is temporary, the effects of the shock on GNE and GDP take between 2 and 5 years to dissipate.

In both models the shock to GNE results in a rise in π_t and r_t, panels (c-d). Ideally, policy would not respond to technology shocks, but it would be very hard for a monetary authority to separate out technology shocks from demand induced rises in GNE. In the current version of H2007 the monetary policy reaction function (represented by the r_t equation) places more emphasis on the GNE gap than inflation - the opposite to what is reported in de Brouwer and Gilbert (2005). In H2007 the associated rise in the real cash rate is similar to that in SVARX2000 (these are shown in panel (e) calculated as the cash response less four times the quarterly inflation response). The real exchange rate appreciates immediately in response to the temporary shock, but, when faced with a permanent shock, ends up with a permanent appreciation, panel (f).

GDP, y_t, shocks

Shocks to GDP are temporary in both models, and have the interpretation of a transitory preference shift to domestic goods over imported ones. As expected from a preference shock, the effect on GDP is greater than the effect on GNE - see Figure 9, panels (b-c). Initially, the effect on π_t is unexpectedly negative, panel (d), while the combination of the effects on π_t and r_t are consistent with an appreciation of the exchange rate, panels (e-f).

Interest rate, r_t, shocks

Shocks to the interest rate are treated as changes in monetary policy. Here a 173 basis point shock is applied. Each of inflation, GNE and GDP shows the anticipated negative response in both H2007 and SVARX2000, see Figure 10, panels (a-c), although there is a small insignificant adverse first quarter responses in H2007. The fall in the inflation rate seems to be of a similar or smaller dimension to that in Buncic and Melecky (2008); see their Figure 2, noting that their shock is somewhat smaller than the one applied here. Higher real interest rates lead to appreciation of the currency, panel (d). Overall, in H2007 the effects of the interest rate shock are smaller for the real variables and inflation than in SVARX2000. The smaller effect in H2007 is presumably a consequence of the equilibrium correction mechanism in the system, and suggests that a levels-based model may overestimate the impact of monetary policy on the economy. In comparison with the results here the Australian SVAR of Berkelmans (2005) reports almost a 1 percent appreciation of the RTWI in response to a 25 basis point interest rate shock. To get a similar response in H2007 would require a shock to r_t of 8 times that size.[15]

5.2 Historical Decomposition of Output

As set out in Appendix B an historical decomposition of a variable utilizes a representation of any variable in terms of the product of its impulses responses with estimates of the structural shocks. From this one can assess the contribution of each shock to the variable over time. H2007 is therefore

[15] Berkelmans (2005) uses a 7 variable VAR with domestic variables represented by GDP, real credit, quarterly inflation, the cash rate and RTWI. The external sector is represented by commodity prices and USGDP.

used to decompose GDP into its component shocks in Figure 11. Each of the contributions is reported on the same scale, which makes it immediately obvious that the most influential of the shocks are technology, preferences and the foreign sector (these being the shocks from the structural equations for ξ_t, y_t, and, foreign GDP and exports). In particular, exports have been contributing positively in more recent years, consistent with the commodity boom. However, the relative price of exports and imports, given by the terms of trade, has not acted to improve output. Part of the explanation for this lies in the offsetting strong appreciation of the domestic currency. The positive effects of booms in the local, q_t, and US, q_t^*, stock markets are clearly evident in the period leading up to 2000, see also Dungey, Fry and Martin (2004).

To understand the impact of monetary policy more clearly, we follow Dungey and Pagan (2000) and construct a monetary policy indicator. This involves suppressing the feedback effects in the interest rate equation, and then adding the difference between those results and those from the original system to the pure effects of the cash rate shocks on the variables in question. The resulting monetary policy indicator is shown in Figure 12(a). The early part of the Figure shows quite substantial volatility, associated with the initial conditions rather than the model results, so it seems wisest to consider analysis after 1986 in order to give the initial conditions time to dissipate.

The results show some differences to those in Dungey and Pagan (2000) and Dungey (2002), where the influence of monetary policy on output was negative from around 1987 until the end of 1993. Here this pattern is interrupted by a short-lived expansionary period for the year from September 1989 to September 1990. The most likely explanation being the effects coming from looser monetary policy after the 1987 stock market crash. From then on policy remained contractionary until December 1993, as found previously. According to the Melbourne Institute business cycle dates the Australian economy peaked in the second quarter of 1990, and troughed in October

1991. Hence the monetary policy stance during the contractionary phase of the cycle was expansionary during the first 6 months, and contractionary thereafter. The contractionary stance of monetary policy continued until 1994 when an expansionary phase occurred until September 1997, whereupon a period of very slight contraction was ushered in over the next 9 months. Following this, monetary policy has been expansionary for output during the remainder of the period, returning to a slightly contractionary stance towards the end of the sample (the end of 2006).

Monetary policy over most of the sample period has been aimed at controlling inflation. Figure 12(b) conducts a similar analysis as for output but now provides a monetary policy indicator for inflation. The figure shows that, from December 2004 onwards, monetary policy has been acting to reduce inflation. For most of the sample period the effects on inflation mirror those upon output. The historical decomposition of this section suggests strong domestic demand contributed to inflation at the end of the sample.

6 Conclusion

This paper explores the relationship between empirically coherent SVAR models of the macroeconomy and models which incorporate theoretical insights from NK models. To do so we use an established empirical SVAR model of the Australian economy in Dungey and Pagan (2000) as a benchmark. It is apparent that the NK models suggest some model designs which are not supported by the data, while the SVAR design suggests some structures which are not present in the NK models. In particular, many empirical implementations of NK models, and our previous SVAR, do not specifically account for permanent and transitory shocks in the data. Hence we explored the impact of building such a feature into the model, as well as allowing for a mix of I(1) and I(0) variables. This led to a new model of the Australian economy, H2007. A comparison with the earlier SVAR, which included only temporary shocks, shows that the inclusion of the longer run relationships suggests that the previous model overstated the impact of interest rate shocks on macroeconomic activity, demonstrating the potential importance of these modelling innovations to policy makers.

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A Data Definitions

Variable	Description	Source
y_t^*	Real US GDP, s.a.	Datastream USGDP...D
s_t	Australian terms of trade	ABS National Accounts
r_t^*	US 90 day interest rate	Datastream USESTBIL quarterly average
π_t^*	US CPI Inflation	Datastream USI64...F
q_t^*	US Q ratio	Datastream Dow Jones USJINDUS divided by the USCPI from
x_t	Real exports, s.a.	Datastream AUEXPGSVD
q_t	Australian Q ratio	All Ordinaries Index Datastream AUSHRPRCF divided by the implicit price deflator for plant and equipment from AUSSTATS A20303942V
ξ_t	Real GNE, s.a.	Datastream AUGNE...D
y_t	Real GDP, s.a.	Datastream AUGDP...D
π_t	CPI inflation	Spliced CPI from ABS with the RBA Aquisitions series for the period when CPI included mortgate interest rate costs.
r_t	11 am cash rate	Datastream AUUOFFL quarterly average
ζ_t	Real TWI	Datastream AUXTW..R quarterly average

B Deriving Impulse Responses and Historical Decompositions

We set out the methodology for a second order system (third order in levels). Let the estimated SVECM(2) system be written as

$$\Delta z_t = \alpha\beta' z_{t-1} + B_0^{-1} B_1 \Delta z_{t-1} + B_0^{-1} B_2 \Delta z_{t-2} + B_0^{-1} \varepsilon_t$$

As in the text we can write this as

$$\Delta\psi_t \;=\; \alpha\beta'\psi_{t-1} + \overline{B}_1\Delta\psi_{t-1} + \overline{B}_2\Delta\psi_{t-2} - \Delta z_t^p + \overline{B}_1\Delta z_{t-1}^p + \overline{B}_2\Delta z_{t-2}^p + B_0^{-1}\varepsilon_t,$$

where $\psi_t = z_t - z_t^p, \overline{B}_j = B_0^{-1}B_j$. Therefore

$$\psi_t = \alpha\beta'\psi_{t-1} + \overline{B}_1\Delta\psi_{t-1} + \overline{B}_2\Delta\psi_{t-2} - J\varepsilon_t + \overline{B}_1 J\varepsilon_{t-1} + \overline{B}_2 J\varepsilon_{t-2} + B_0^{-1}\varepsilon_t,$$

since $\Delta z_t^p = J\varepsilon_t$, where $J = \beta_\perp(\alpha'_\perp\Gamma\beta_\perp)^{-1}\alpha'_\perp B_0^{-1}$. This provides a VARMA system in the $I(0)$ variables ψ_t which can be written as

$$G(L)\psi_t = H(L)\varepsilon_t,$$

showing that the impulse responses are $G(L)^{-1}H(L)$. These can be computed in the normal way for a stationary system.

Once we have the impulse responses $\frac{\partial\psi_{t+j}}{\partial\varepsilon_t}$ we can find those for the original variables z_t using

$$\begin{aligned} C_j &= \frac{\partial z_{t+j}}{\partial\varepsilon_t} = \frac{\partial\psi_{t+j}}{\partial\varepsilon_t} + \frac{\partial z_{t+j}^p}{\partial\varepsilon_t} \\ &= \frac{\partial\psi_{t+j}}{\partial\varepsilon_t} + J. \end{aligned}$$

Historical decompositions can be derived by simply recognising that the VARMA form allows for any variable to be written as a weighted sum of previous shocks plus the effects of an an initial condition i.e.

$$\Delta z_t = \textit{initial conditions} + \sum_{i=0}^{t} C_i\varepsilon_{t-i}, \tag{24}$$

where C_i represent the impulse response functions.