Intertrade durations in US Treasuries: a FIACD model*

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Abstract

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1 Introduction

The time interval between successive transactions in a financial market conveys information about the potential liquidity effects. This effect differs from the more commonly studied price impact which determines how much can be traded without disrupting prices. Intertrade duration is an indication of the ability to trade at any price. It is already known that intertrade duration displays persistence and time clustering, leading to the development of the autoregressive conditional duration (ACD) models of Engle and Russell (1998) and further developments in Bauwens and Giot (2000), Zhang, Russell and Tsay (2001) and Bauwens and Hautsch (2006) particularly. These models treat the trade interval as having either unit root or stationarity properties. Practically, this means that shocks to duration are treated as either instantly absorbed or permanent. In reality, the observed behaviour of the data suggests the truth lies somewhere between these two extremes. Shocks to duration, caused perhaps by information arrival, may have longer run, but not permanent, effects on future duration. Jasiak (1999) develops the concept of the fractionally integrated ACD $(FI - ACD)$ model for this case, and applies it to Alcatel and IBM stocks.

In this paper we develop the $FI - ACD(p, d, q)$ model of Jasiak and apply it to the US Treasury market. The US Treasury market is arguably the most important financial market in the world, in 2008 it remains the largest single market by turnover volume. Its microstructure has not been significantly explored, mainly due to difficulties with databases. In recent years the market has transited from a voice-over protocol to Electronic Communication Netwrok (ECN), see Mizrach and Neely (2006). Previous studies on the voice-over network include Fleming and Remolona (1999), Green (2004) and Brandt and Kavajecz (2004) *inter alia*). Empirical work concerning the more recent ECN data is only beginning to emerge, see Mizrach and Neely (2006,2007), Jiang et al (2007), Dungey, McKenzie and Smith (2008). Durations in this market have not previously been studied.

Since 2000 US bond market trading has been dominated by the ECNs of Cantor Fitzgerald and ICAP. The eSpeed (Cantor Fitzgerald) and BrokerTec (ICAP) databases provide trading information for each of these markets. Mizrach and Neely (2006) find that there are qualitatively few differences between the two. The dataset for the current paper is drawn from the eSpeed database. The most obvious advantage of the new ECN datasets over the previous voice-over protocol data provided in GovPX is that trades are accurately tracked and identified. The eSpeed dataset represents 10 millisecond shots of the transaction process, which is the maximum updating frequency that the traders using this platform see in real time. The focus here is on the time taken between intiations of successive transactions, the trade duration, for trading in the on-the-run Treasuries of the 2 year benchmark bonds. The sample period is January 3, 2006 to October 10, 2006, representing some 116,479 observations. We restrict our attention to the most active part of the trading day, from 7:30am to 5:30pm New York time. On average each day in the sample contains 623 trades worth $US21 billion.

The concept of duration in this market compatible with those used in others is the time between the intiation of consecutive trades.Intradaily data is typically characterised by strong diurnality (see Engle and Russell, 2004), that may bias any estimation results. We follow the approach of, *inter alia*, Engle and Russell 1998, 2004 and Zhang, Russell and Tsay (2001) by constructing diurnalised estimates of duration and workup times. Defining the raw duration between the i^{th} and $i-1^{th}$ transactions as $x_i = t_i - t_{i-1}$, then the adjusted duration is:

$$x_i^* = \frac{x_i}{\Phi\left(t_{i-1}\right)}, \tag{1}$$

The deterministic effect on trade durations due to the time of day is defined as the expected duration conditioned on time-of-day $\Phi\left(t_{i-1}\right) = E\left(x_i | t_{i-1}\right)$. This expectation is obtained by averaging the durations over thirty minutes intervals for the trading day. A cubic spline is employed to smooth the time of day function across the thirty minutes intervals.By construction, the mean of x_i^* is appoximately 1.

A summary of these diurnalised trading intensity measures is also presented in Table1. ncluding the number of trades (n), sample average (μ), p-value for a test of the null of a sample average of zero (in parenthesis) and standard deviation (σ^2). The results of a battery of Ljung-Box tests for p^{th} order serial correlation in x_i and the corresponding squares are also presented in Table 1 and the results uniformly reject the null hypothesis of no serial correlation. The results show considerable structure in x_i^*, the adjusted durations. The autocorrelation functions (ACF) for the data in Figure 1 display the slow decay which motivates the use of the FI-ACD(p, d, q) approach.

2 Duration Modelling

The ACD model introduced by Engle and Russell (1998) is designed to capture the clustering and serial correlation in the duration between trades. The ACD (p,q) is given by

$$
\begin{aligned}
x_i^* &= \psi_i \varepsilon_i, \\
\psi_i &= \mu + \gamma(L) x_i^* + \omega(L) \psi_i
\end{aligned}
\tag{2}
$$

where ψ_i is the expected value of duration given the information set, the error term ε_i,follows some defined process, $\gamma(L) = \gamma_1 L + \gamma_2 L^2 + \ldots + \gamma_p L^p$ and $\omega(L) = \omega_1 L + \omega_2 L^2 + \ldots + \omega_q L^q$. In many markets an ACD(1,1) with exponential or Weibull error distribution has been found to be a good representation of the data (see Hautsch 2006 for example).

The expected duration can be expressed as an infinite-order process

$$\psi_i = [1 - \omega(L)]^{-1} \mu + [1 - \omega(L)]^{-1} \gamma(L) x_i^* \tag{3}$$

and defining $v_i = x_i^* - \psi_i$ this is conveniently expressed as

$$[1 - \gamma(L) - \omega(L)]\, x_i^* = \mu + [1 - \omega(L)]\, v_i \tag{4}$$

or equivalently defining $\phi(L) = [1 - \gamma(L) - \omega(L)]^{-1}$

$$\phi(L)(1 - L) x_i^* = \mu + [1 - \omega(L)]\, v_i \tag{5}$$

Analagously to the development of FIGARCH in Baillie, Bollerslev and Mikkelsen (1996), the FI-ACD model is simply a replacement of the $(1-L)$ operator in (5) with a fractionally integrated operator $(1-L)^d$. The representation of interest for estimation is

$$[1 - \omega(L)]\psi_i = [1 - \omega(L)]^{-1} \mu + \{1 - [1 - \omega(L)]^{-1} \phi(L)(1 - L)^d\} x_i^* \tag{6}$$

Table 2 presents quasi-maximum likelihood estimates of the $ACD(1,1)$ and the $FI-ACD(p,d,q)$ for lag orders $p, q = 1, 2$, asssuming an exponential distribution for ε_i.

Conditions to ensure the positivity of the expected duration will be analagous to those for FIGARCH. Recently, Conrad and Haag (2006) have developed necessary and sufficient conditions for the cases where $q \leq 2$. These involve inequality constraints on combinations of the estimated parameters. Each of the FI-ACD(p, d, q) models presented in Table 2 satsify the parameter restrictions to support a positive covariance matrix (the relevant cases are Theorem 1, Case 1 for the FI-ACD$(1, d, q)$ models and Theorem 2, Case 2 for the FI-ACD$(2, d, q)$, see Conrad and Haag 2006 for the full set of potential cases).

The results in Table 2 are instructive in that they illustrate first, just how much of the structure in the durations is captured by the $ACD(1, 1)$. However, any parameterisation that allows for long memory represents a more appropriate conditional characterisation of the durations. The estimated fractional differencing parameter $\hat{d}$ is statistically significantly different to zero for all $FI - ACD(p, d, q)$ models considered. This result suggests that the ACD(1,1) represents a potentially misspecified conditional charaterisation of the duration data. For our preferred model, the $FI - ACD(2, d, 2)$, despite a sample size of excess of 116,000 observations there is little evidence at the 1 percent level of remaining correlation in the data. The first 10 serial correlation coefficients from the $FI - ACD(2, d, 2)$ model are {-0.0048,-0.0067,-0.0015,-0.0002,0.0014,-0.0003,-0.0035,-0.0089,-0.0001,-0.0075} all of which are very close to zero in magnitude. These results suggest that there is long memory in shocks to trade durations in the 2 year US Treasury bond, which can be parsimoniously captured by an $FI - ACD(2,d, 2)$ model.

3 Summary and Conclusion

Unlike the equity and foreign exchange markets, there is relative little existing research into trading in fixed income markets. This paper examines a sample of 116,479 durations between trades in US Treasury Bonds with 2 year maturity sampled over the period January 3, 2006 to October 10, 2006. We find strong evidence of persistence in these durations. In contrast to the equity and foreign exchange markets, an $ACD(1, 1)$ model is not sufficient to capture all of the structure in these data. The evidence suggests that an $FI - ACD(2,d, 2)$ model provides a parsimonious condtional characterisation of the data.

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Table 1: Data Descritpion

Diurnalized Durations				
n	μ	σ^2	$Q\,(10)$	$Q\,(20)$
$116,479$	1.1414	5.3739	6176.3812	7792.5831
	[0.0000]		[0.0000]	[0.0000]

Table 2: FI-ACD Estimates

	$(1,0,1)$	$(1,d,0)$	$(1,d,1)$	$(2,d,1)$	$(2,d,2)$
μ	0.0067	0.0054	0.0033	0.0000	0.0000
	(0.0009)	(0.0029)	(0.0012)	(0.0001)	(0.0001)
γ_1	0.9874	$-$	0.4864	0.4977	0.5527
	(0.0020)		(0.0266)	(0.0227)	(0.0201)
γ_2	$-$	$-$	$-$	$-$	0.0456
					(0.0104)
ω_1	0.9398	0.0982	0.6673	0.6678	0.7687
	(0.0051)	(0.0094)	(0.0249)	(0.0252)	(0.0228)
ω_2	$-$	$-$		0.0087	0.0064
				(0.0023)	(0.0021)
d	$-$	0.2119	0.3095	0.3004	0.3355
		(0.0063)	(0.0154)	(0.0095)	(0.0165)
$Q\,(10)$	145.86	61.407	36.959	37.230	25.549
	[0.0000]	[0.0000]	[0.0001]	[0.0001]	[0.0044]
$Q\,(20)$	188.958	86.806	44.985	45.521	37.555
	[0.0000]	[0.0000]	[0.0011]	[0.0009]	[0.0100]

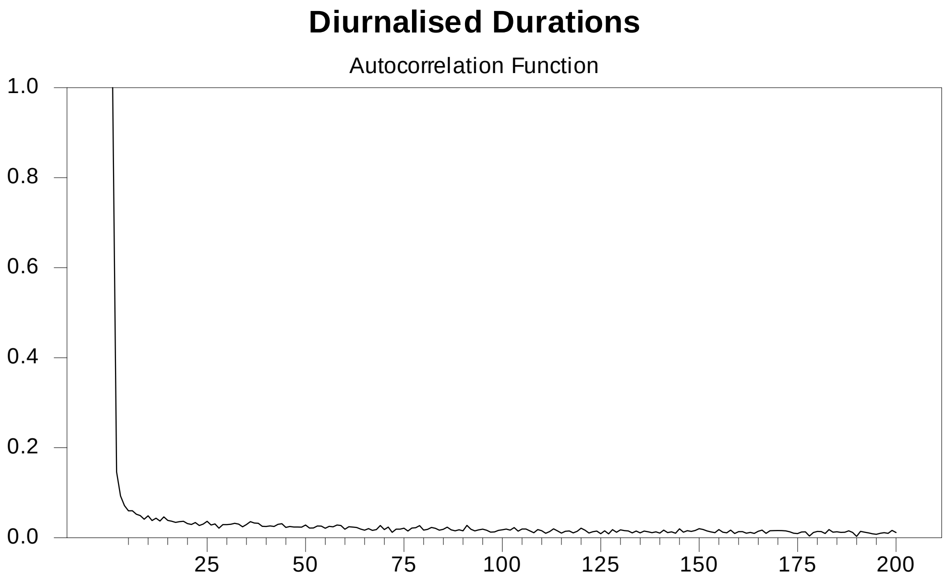

Figure 1: Autocorrelation Function for x_i^*.